JET BOUNDARY AND FREE SURFACE BEHIND A SOLID OF REVOLUTION UNDER A LONGITUDINAL PRESSURE GRADIENT

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Described is a method of calculating the ideal boundary of a supersonic underexpanded jet and the free surface behind a solid of revolution. The results of calculations are compared with those obtained by the exact method of characteristics.

In order to calculate the mixing of a supersonic stream with the ambient gas, it is necessary to know the location of the ideal stream boundary. This boundary, as well as the entire flow field, can be calculated by the method of characteristics, applicable to either the internal or the external problem. Whenever only the stream boundary needs to be known, it is reasonable to use approximate methods of calculation.

During the discharge of a supersonic underexpanded jet into a moving medium, under conditions applicable to the internal problem, there appears a longitudinal pressure gradient which affects the location of the ideal jet boundary. In the external problem with an appreciable backstream, analogously, the longitudinal pressure gradient deforms the ideal free surface behind the bottom segment of a solid of revolution.

The ideal jet boundary and free surface, either under constant pressure or under a longitudinal pressure gradient, can be found on the basis of the hypothesis proposed in [1]. The jet boundary under conditions with a zero pressure gradient has already been calculated in [2].

According to [1], at every point on the jet boundary there take place two processes: rarefaction and compression. The former is characterized by a quasi one-dimensional increase in the jet area by an amount dF_i, the latter is characterized by the rotation of the stream through an angle $d\vartheta$ and the formation of a weak compression wave. The following equation applies along the jet boundary:

$$\frac{\partial \bar{p}}{\partial \bar{F}_i} dF_i + \frac{\partial \bar{p}}{\partial \vartheta} d\vartheta = d\bar{p}. \tag{1}$$

From the relations for an isentropic flow follows

$$\frac{\partial \bar{p}}{\partial \bar{F}_i} = -\frac{k\bar{p}M_i^2}{M_i^2-1} \frac{q(M_i)}{q(M_a)}, \tag{2}$$

and the linearized theory yields

$$\frac{\partial \bar{p}}{\partial \vartheta} = \mp \frac{k\bar{p}M_i^2}{M_i^2-1}. \tag{3}$$

(Here and henceforth, whenever two signs appear before an expression, the upper sign refers to the internal problem and the lower sign refers to the external problem.)

At the jet boundary ($\gamma = 1$) and at the free surface ($\gamma = 0$) we have

$$d\bar{F}_i = (1+\gamma)\bar{r}_i^{\gamma} d\bar{r}_i. \tag{4}$$

Furthermore, obviously

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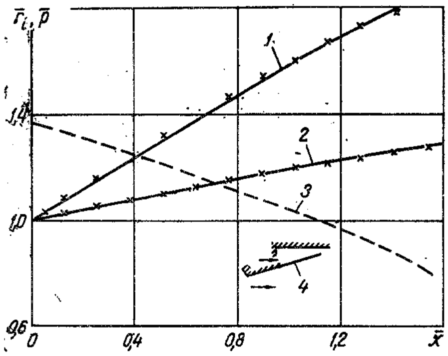

Fig. 1

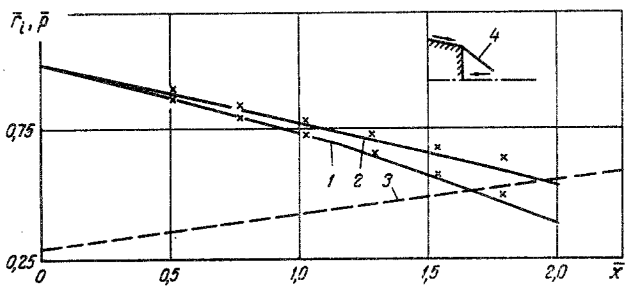

Fig. 2

Fig. 1. Boundary of a supersonic underexpanded jet: 1) $M_a = 1.6$, $\bar p = 0.1158$, $\alpha = 0$ [3]: 2) $M_a = 5.14$, $\alpha = 15°$ [4]; 3) pressure distribution along the jet boundary in the case 2); 4) jet boundary; curves represent the solution by the method of characteristics; dots represent the solution to system (6).

Fig. 2. Free surface behind a solid of revolution: 1) $M_a = 3$, $\bar p = 0.3$, $\alpha = 0$; 2) $M_a = 3$, $\alpha = 0$; 3) pressure distribution along the free surface in the case 2); 4) free surface; curves represent the solution by the method of characteristics; dots represent the solution to system (6).

$$\frac{d\bar r_i}{d\bar x} = \mathrm{tg}\,\vartheta. \tag{5}$$

Inserting (2)-(4) into (1) and then dividing by $d\bar x$ yields

$$\pm\frac{d\vartheta}{d\bar x} + \frac{1+\gamma}{\sqrt{M_i^2-1}}\cdot\frac{q(M_i)}{q(M_a)}\,\bar r_i^{\gamma}\,\frac{d\bar r_i}{d\bar x} + \frac{\sqrt{M_i^2-1}}{k\bar p M_i^2}\cdot\frac{d\bar p}{d\bar x} = 0,$$
$$\frac{d\bar r_i}{d\bar x} = \mathrm{tg}\,\vartheta. \tag{6}$$

The solution to system (6) for the initial conditions

$$\bar x_0 = 0,\quad \bar r_{i0} = 1,\quad \pm(\vartheta_0 - \alpha) = \omega(M_{i0}) - \omega(M_a)$$

determines the location of the jet boundary or of the free surface, at a given pressure gradient $d\bar p/d\bar x$. The relation between pressure and the Mach number M_i can be expressed as

$$\bar p = \left(\frac{1+\frac{k-1}{2}M_a^2}{1+\frac{k-1}{2}M_i^2}\right)^{\frac{k}{k-1}} \tag{7}$$

In the special case without a pressure gradient, system (6) becomes

$$\pm d\vartheta + \frac{1+\gamma}{\sqrt{M_i^2-1}}\cdot\frac{q(M_i)}{q(M_a)}\,\bar r_i^{\gamma}\,d\bar r_i = 0,$$
$$\frac{d\bar r_i}{d\bar x} = \mathrm{tg}\,\vartheta. \tag{8}$$

Since now M_i = const, according to (7), hence integration of system (8) yields

$$\bar x = \int_1^{\bar r_i} \mathrm{ctg}\left[\vartheta_0 \pm \frac{1}{\sqrt{M_i^2-1}}\,\frac{q(M_i)}{q(M_a)}\,(1-\bar r_i^{1+\gamma})\right] d\bar r_i. \tag{9}$$

System (6) was numerically integrated on a computer by the Runge–Kutta method. The results were then compared with those obtained by the method of characteristics, as shown in Figs. 1 and 2.

In Fig. 1 are shown the calculations for the internal problem. The results based on the method of characteristics were taken from [3] and [4]. In Fig. 2 are shown the results for the external problem. A. S. Silina assisted in the calculation made by the method of characteristics.

Both graphs indicate that the described approximate method of calculating the jet boundary and the free surface behind a solid of revolution under a longitudinal pressure gradient yields results which are in close agreement with the results of exact calculations by the method of characteristics.

NOTATION

M is the Mach number;
$\bar{p} = p/p_a$ is the pressure at the jet boundary (or the free surface);
$\bar{F} = F/F_a$ is the cross section area of a boundary stream tube;
$\bar{x} = x/r_a$ is the distance from the nozzle throat (or from the solid of revolution);
$\bar{r} = r/r_a$ is the distance from the flow axis to the jet boundary (or the free surface);
k is the adiabatic exponent;
α is half the nozzle angle (slope angle of the generatrix of the solid of revolution);
$q(M) = M\{(k+1)/[2+(k-1)M^2]\}^{(k+1)/2(k-1)}$;
$\omega(M) = \sqrt{(k+1)/(k-1)}\ \mathrm{arctg}\sqrt{(k-1)(M^2-1)/(k+1)} - \mathrm{arctg}\sqrt{m^2-1}$.

Subscripts

a refers to nozzle throat (bottom segment of the solid of revolution);
i refers to the jet boundary (or the free surface).

LITERATURE CITED

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